Technical Comments

Comment on "Thermally Induced Vibration and Flutter of Flexible Booms"

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IN his analysis of bending oscillations of flexible booms subject to radiant (solar) heat, Yu reaches the conclusion that, in the absence of damping, "(the motion of) the boom is stable if it is pointed away from the sun and unstable if towards the sun."[1] This conclusion has been strongly discussed. Although some of the earliest criticisms offered were manifestly misdirected (for example, in Ref. 2; cf. the second "final remark" in Ref. 4), it seems conclusively proven now that the introduction of an inconsistent boundary condition in Yu's analysis led to a reversal in the "sign" of the conclusion, which should read therefore, "the boom is stable if pointed towards the sun, etc."[3,4]

This corrected conclusion is also supported by an immediate extension of the results that had been obtained earlier by this writer,[5] with reference to two model booms in which all deformations (elastic, viscoelastic, and thermal) were lumped in one or two cells of vanishing length a (Figs. 1 and 2).† The study of very simple theoretical models of this type offers the possibility of developing perfectly consistent, exact arguments. Thus, if the model is well chosen, useful qualitative indications can be obtained in cases like the present one, in which the treatment of more realistic models can only be pursued in an approximate fashion and it is difficult to evaluate the effect of the approximations.

Included in Ref. 5 is the possibility of an axial load P, which for the second model could either be constantly parallel to the initial boom axis x or follow the rotation of the boom tip (Fig. 2). The axes of the booms were assumed to be parallel to the direction of radiation, i.e., with the notation in Figs. 1 and 2, $\alpha = \pm 90°$, the case $\alpha = +90°$ being characterized by negative values of the parameter K^*, which measured the intensity of the heat source.‡ The relationship between the thermal angle of rotation in a cell ϕ_{iT}, the total rotation and the heat intensity K^*, derived in Ref. 5 for a sandwich section, coincides essentially with that proposed earlier by Etkin and Hughes for a tubular section[7] and with Yu's Eq. (6).

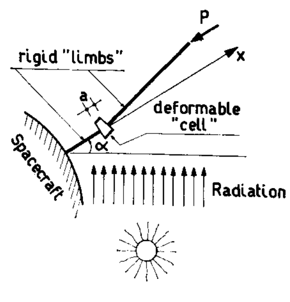

Fig. 1 One-degree-of-freedom boom model.

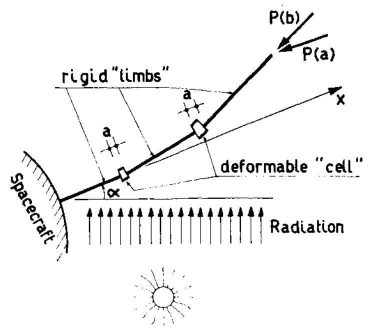

Fig. 2 Two-degree-of-freedom boom model.

The equations of small oscillations of the models about the equilibrium position yielded, respectively, a third-order homogeneous linear differential equation and a set of two such equations. In both cases, the general solution can be written in the form

$$\phi_{iT} = \Sigma_j A_j \exp (s_j t) \qquad (1)$$

where s_j are the solutions of an algebraic equation. The motion is stable when all s_j have negative real parts. Assuming that the s_j vary continuously with the relevant parameters,

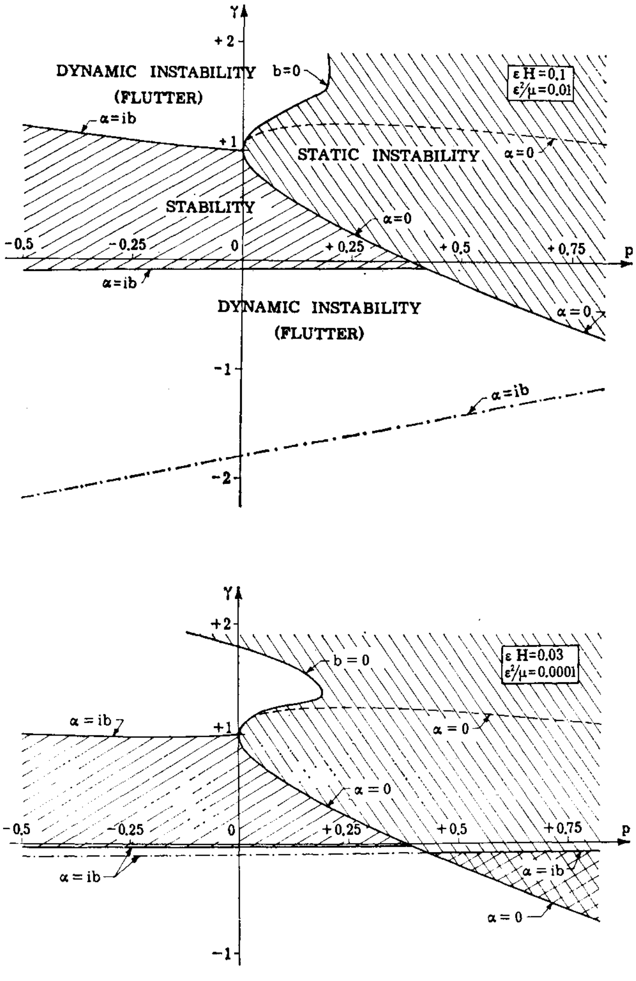

Fig. 3 Examples of stability and instability regions for boom in Fig. 2, under unidirectional load P (a).

Received August 4, 1969; revision received October 9, 1969. This publication was supported by a C.N.R. (Italian National Research Council) Contribution.

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† The contradiction between the writer's and Yu's results was first pointed out by Beam.[6]

‡ The symbols K and α_j of Ref. 5 are here indicated by K^* and s_j in order to avoid confusion with Yu's nomenclature, which is used in the present Note as far as possible.

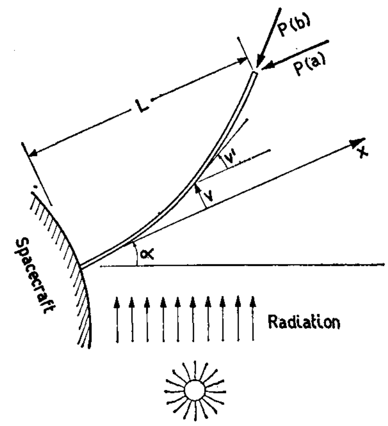

Fig. 4 Fully deformable cantilever boom.

the boundaries of stability can therefore be obtained by imposing that one s_j is either zero or purely imaginary (corresponding to a static, buckled position of equilibrium and to a purely harmonic mode of oscillation). Such boundaries were numerically determined in a number of cases. Two typical results (pertaining to the two-cell model with unidirectional load P) are shown in Fig. 3, where p, ϵH and μ are dimensionless measures of the load P (positive in compression), the internal viscous damping and the boom mass, respectively; and

$$\gamma = \alpha_T K^* a/Hh = Ka/H \tag{2}$$

is a measure of the intensity of radiation. In Eq. (2), α_T is the coefficient of thermal expansion, h the depth of the cell cross section, and H an emission parameter, related to Yu's characteristic time τ by

$$H = 1/\tau \tag{3}$$

It is of interest to note that, as could be expected, the boundary of static instability ($s = 0$) is independent of the damping ϵ, while the lower boundary of dynamic instability ($s = ib$) tends to the p axis as $\epsilon \to 0$.

If $\alpha = \text{const} \neq \pm 90°$, the above summarized analysis remains valid, provided the definition of γ is modified into

$$\gamma = (Ka/H)(-\sin\alpha) \tag{4}$$

with K^* and K always non-negative, so that $\text{sgn}(\gamma) = \text{sgn}(-\sin\alpha)$. Thus, the equations of small oscillations are modified only because constant terms, proportional to $\gamma(-\cot\alpha)$, appear on the right-hand side. This means that the oscillations occur about a deformed position of equilibrium, rather than about the straight, original configuration, but their stability does not change. All results as to stability regions obtained in Ref. 5 (of which Fig. 3 is an example) hold for $\alpha \neq \pm 90°$, with the obvious observation that the boundary of static instability $s = 0$ corresponds now to infinitely large equilibrium deformations (exactly as it happens for an eccentrically loaded elastic column at the critical load value).

The complete extension of the previous analysis to the fully deformable boom in Fig. 4 is now under study, but computational difficulties have so far prevented numerical results. to be obtained. However, the similarity to be expected with the results obtained earlier[5] with reference to the proposed models can be easily inferred from a direct determination and comparison of two boundaries of stability in the $p - \gamma_1$ plane, where

$$\gamma_1 = (KL/2H)(-\sin\alpha) \tag{5}$$

$$p = PL^2/EI \tag{6}$$

First, in the absence of damping, the p axis is a boundary of dynamic instability, because when $\gamma_1 = 0$, the motion of the boom about its position of equilibrium is the sum of purely harmonic modes $A_j e^{ib}$.

The position of equilibrium is determined by

$$EI(v'' - \kappa_T) + M = 0 \tag{7}$$

where κ_T is the thermal curvature

$$\kappa_T = (K/H)\cos\alpha - v'(K/H)\sin\alpha \tag{8}$$

and M the bending moment

$$M = -P[v(L) - v(x)] \quad \text{(unidirectional force)} \tag{9a}$$

or

$$M = P[(L - x)v'(L) - v(L) + v(x)] \quad \text{(follower force)} \tag{9b}$$

In the case of unidirectional force, the (static) boundary conditions are

$$\begin{aligned} v(0) &= 0 \\ v'(0) &= 0 \\ v''(L) - \kappa_T(L) &= 0 \\ v'''(L) - \kappa_T'(L) &= -Pv'(L)/EI \end{aligned} \tag{10}$$

In case of follower force, the fourth Eq. (10) becomes

$$v'''(L) - \kappa_T'(L) = 0 \tag{11}$$

Equations (5–11), after some algebra, yield the equations of the boundary of static instability (i.e., the $p - \gamma_1$ curves that correspond to indeterminate deflections v if $\alpha = \pm 90°$, to infinitely large v otherwise) [see Fig. 5]:

a) unidirectional force

$$\gamma_1 = u(e^u + e^{-u})/(e^u - e^{-u}) \tag{12}$$

b) follower force

$$\gamma_1 = 2ue^{\gamma_1}/(e^u - e^{-u}) \tag{13}$$

where

$$u = (\gamma_1^2 - p)^{1/2} \tag{14}$$

Equations (12) and (13) are plotted in Figs. 5a and 5b, respectively. It can be noticed that they are qualitatively similar to those obtained in Ref. 5 for the two-cell model (for example, cf. Fig. 5a and Fig. 3 here).§ It can therefore be inferred that the region between the curve, Eq. (12) or Eq. (13), and the p axis is a stable region. Thus, in absence of axial load ($p = 0$), which is the only case considered by Yu and the other quoted researchers,[3,4,7] the boom is stable when $\gamma_1 > 0$ (or $\sin\alpha < 0$); i.e., "if pointed towards the sun," at least within a certain

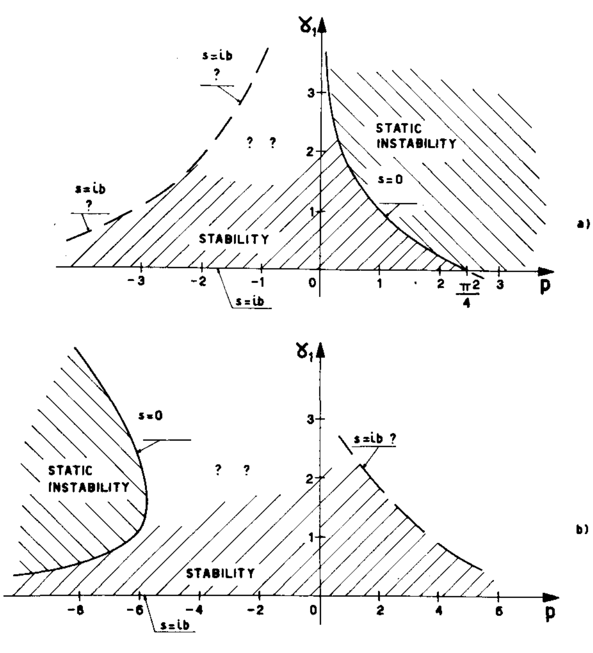

Fig. 5 Boundaries of stability for boom in Fig. 4: a) unidirectional force; b) follower force.

§ By analogy, it seems likely that the full dynamic analysis would give another boundary of dynamic stability, as sketched in a purely indicative way in Fig. 5 (dashed line).

range of γ_1. The boom is unstable (if the damping is negligible) for any $\gamma_1 < 0$ (or $\sin \alpha < 0$), i.e., "if pointed away from the sun."

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Comment on "Thermally Induced Vibration and Flutter of a Flexible Boom"

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THERE is a fallacious argument in the paper on thermal bending flutter of long flexible booms by Yi-Yuan Yu.[1] In consequence, the main result[1] is just the reverse of the correct answer.

Thermal flutter is readily accessible to physical intuition. We will clarify this statement first, and will then identify the error in Ref. 1.

From the point of view of the theory of dynamic stability,† the thermal flutter of booms corresponds to the aerodynamic flutter of airplane wings. In both, there is an elastic structure imbedded in a continuous stream of energy (of flowing air or of thermal radiation). There are oscillatory reactions from the stream onto the structure, which arise when the structure undergoes some oscillatory deformation (started by some incidental disturbance).

The mechanism which most often causes aerodynamic flutter is coalescence of the frequencies of two different degrees of freedom. However, large reaction forces are required to make two frequencies coalesce, and the thermal reactions which occur in space may be relatively weak. It thus makes sense to look at the case of a boom which oscillates in single degrees-of-freedom bending and is exposed to low-intensity thermal radiation. In essence, this is what Yu[1] sets out to do.

Whether the effect of the thermal field will be stabilizing ("damping"; oscillatory structural energy is lost) or destabilizing ("flutter"; oscillatory energy is extracted by the structure from the continuous field of thermal energy) depends upon the product of two signs, 1) the direction of the reactive effects, and 2) the phase (time) relation between reactive effect and structural oscillation.‡

Here one can refer to the familiar analytical concept of structural damping. The structural damping reaction has the same direction as the elastic reaction (structural damping is "restoring") but it is *advanced* in phase. Hence, we know that this combination of the two signs causes loss of energy, damping of the oscillatory motion.

The thermal reaction is *delayed* in phase. Thus we come to a simple rule R for thermal dynamic stability:

"A 'restoring' thermal reaction (one which is stabilizing in the static situation) will tend to produce flutter (will be destabilizing in the dynamic situation), and vice versa."

Applied to the problem of thermal bending flutter, R predicts: 1) the boom will be prone to flutter if it is directed away from the sun (in this case the thermal effect is restoring: to deflect the boom will cause a change in the thermal input such that the deflection is reduced); and 2) the boom will not flutter if it is directed toward the sun (in this case there is a tendency towards static thermal divergence).

These predictions regarding thermal bending flutter agree with the analytical results of G. Augusti.[4]§ Applied to the problem of thermal torsional flutter of open booms, R predicts flutter if the thermal radiation is directed toward the slit. This prediction is beautifully confirmed by the experiments of R. M. Beam.[7]

In contrast to R, Yu[1] states: "(Bending) motion is stable if the boom is pointed away from the sun and unstable if towards the sun." We next identify the fallacy in the analysis of Ref. 1 which led Yu to his erroneous statement.

We start with the here required terms of Eq. (2) or Eq. (3) of Ref. 1. These terms are listed on the right-hand side of the following relation:

$$\int_0^l M_T w'' dx = \int_0^l M_T'' w dx + [M_T w' - M_T' w]_0^l \quad (1)$$

Details which are here unnecessary have been left out in Eq. (1).

The integral on the left hand side of Eq. (1) arises when one writes the work done by the thermal moment M_T, namely, its product with the change in the beam curvature, w''. The right-hand side is the result of integrating twice by parts.

From Eqs. (5) and (7) of Ref. 1

$$M_T = EI[-k_c + k_s(w' - \tau \dot{w}')] \quad (2)$$

Since we are only concerned with the question of stability, we are interested only in out-of-phase terms, and can disregard k_c. For the remainder we use the following as a tracer

$$k_s w' \text{ traces } M_T \quad (3)$$

The tracer terms in Eq. (1) are hence

$$k_s \int_0^l w' w'' dx = k_s \left[\int_0^l w''' w dx + (w'^2 - w w'')_0^l \right] \quad (4)$$

The terms on the right-hand side of Eq. (4) should appear in Eq. (10) of Ref. 1. However, having informed the reader that to leave out the end terms "simplifies greatly," Yu retains only the integral; thus

$$\left(\int_0^l M_T w'' dx \right)_{\text{tracer}} = k_s \int_0^l w' w'' dx \stackrel{?}{=} k_s \int_0^l w''' w dx\,\P \quad (5)$$

Let us see how well this simplification is justified.

Received September 4, 1969; revision received August 25, 1970.

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† "Dynamic stability" is here used in its natural meaning, N. J. Hoff,[2] rather than in the restrictive sense of V. V. Bolotin.

‡ In 1929, H. Glauert[3] showed that energy extraction is possible in principle by showing that single degree-of-freedom flutter of airplane wings is a physical possibility even in the fully linearized formulation of the aerodynamic problem.

§ The writer much regrets that he was unaware of this pioneering work[4] when he wrote his own paper, Ref. 5 and welcomes the fact that Professor Augusti has now undertaken[6] to familiarize the readers of the *Journal of Spacecraft and Rockets* with his results.

¶ Yu,[1] Eq. (10).